Implementation of REED MULLER CODE in MATLAB

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Abstract— In communication error is the most affected part. The receiver receives the original signal with error signal. When the system knows how much error has come with the original signal than an error will remove. So much research has been done in error correction and detection of incoming signals. So in this paper examines the family of codes known as Reed-Muller codes. The code is developed by hardened code and give the brief introduction of the codes and their history before diving in and examining how they are used in practice in Matlab. Design some representation of generator matrices for these codes with the help of Hadamard code, and then discuss the encoding process.

Index Terms— Matlab Simulation tool

1. Introduction

Reed - Muller codes are amongst the oldest and most well-known of codes. They were discovered and proposed by D. E. Muller and I. S. Reed in 1954. Reed-Muller codes are some of the oldest error correcting codes. Error correcting codes are very useful in sending information over long distances or through channels where errors might occur in the message. They have become more prevalent as telecommunications have expanded and developed a use for codes that can self-correct. Reed-Muller codes have many interesting properties that are worth examining; they form an infinite family of codes, and larger Reed-Muller codes can be constructed from smaller ones.

Unfortunately, Reed-Muller codes become weaker as their length increases. However, they are often used as building blocks in other codes. One of the major advantages of Reed-Muller codes are their relative simplicity to encode messages and decode received transmissions. Reed-Muller codes, like many other codes, have tight links to design theory; we briefly investigate this link between Reed-Muller codes and the designs resulting from affine geometries.

2. Review of Previous work

In sec [7] Reed Muller code is generated by Vectors manipulated by three main operations: addition, multiplication, and the dot product. In the previous Reed Muller code is generated by a vector or scalar operation base but this paper code is generated by Hadamard code.

In previous papers the programming language of message encoding at the transmitter end and error decoding at the receiver end in C, but in this research the programming of generating matrix, encoded message from the transmission end in the form for Matlab simulation.

In the future progress in this research, decode the error at the receiver end.

The approach to error correction coding taken by modern digital communication systems started in the late 1940's with the groundbreaking work of Shannon, Hamming and Golay. Reed-Muller (RM) codes were an important step beyond the Hamming and Golay codes because they allowed more flexibility in the size of the code word and the number of correctable errors per code word. Whereas the Hamming and Golay codes were specific codes with particular values of q; n; k; and t, the RM codes were a class of binary codes with a wide range of allowable design parameters. Binary Reed-Muller codes are among the most prominent families of codes in coding theory. They have been extensively studied and employed for practical applications. In this research, I will try to do the performance simulation of Reed-Muller Codec. An introduction on Reed-Muller codes, were introduced that consists of defining the key terms and operation used with the binary numbers.
Defined the Reed-Muller codes and its encoded matrices were. The simulation of the decoding part will also try to show in this research. The performance of Reed-Muller codes were analyzed in terms of its code rate, code length and the minimum Hamming distance.

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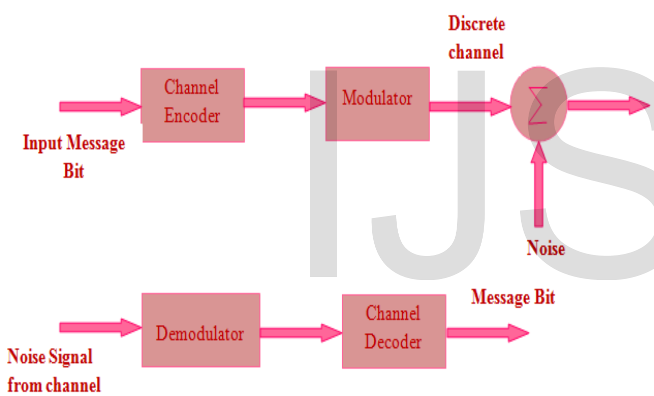
3. Justification and Need for the System

The transmission process, the transmitted signal passes through some noise channel. Due to noise interference, some errors are introduced in the received data. These errors can be detected and sometime corrected using coding techniques. Generally the error control methods are of two types;

1) Error Detection with Retransmission
2) Forward acting error correction.

In the first method, when an error is detected, the retransmission request (AGQ) is sent back to the transmitter. In the second method, the error is detected and corrected by proper coding techniques at the receiver end. Whenever a single source transmits data to the number of receivers, the forward acting error correction is used. This is due to the fact that the retransmission is impossible in this case.



Error coding techniques play the important role in modern digital communication, it detects the error from the transmitted signal and collect the same data at the receiver end.

In Matlab simulation tool there are lots of error control techniques like linear block code, cyclic code, convolution code and hamming code but there is no space for Reed muller code so the main aim of this research is to generate a program in Matlab format and encode and detect the error in transmitting signals by MATLAB simulator.

4. Definition of Terms and Operations

This paper consists of strings of length 2^m, where m is a positive integer. The codeword of a Reed Muller code forms a subspace of such a space. Vectors can be manipulated by three main logic operations: permutation combination and binary logic operations.

Code Word: The encoded block of 'n' bit is known as a code word. It consist of message bit and redundant bits.

Block Length: The number of bit 'n' after coding is known as the block length of code.

Code rate : The ratio of message bit 'm' and encoder output bits 'n' are known as code rate

$r = k / n$ we find that $o < r < 1$

Simple view of Reed-Muller Codes: We now investigate the basics of Reed-Muller codes, including what they are, and a technique for encoding and decoding. The design theory of Reed-Muller (r, m) codes is depending upon the following steps.

- Calculate the number of rows of generate matrix by the permutation and commutation.

  R = m Cr-n => R = !m / (!r !(m-r)

- Calculate the number of Colum of generate matrix.

  $\mathbf{M = 2^m}$

Example: Calculate the row and Colum for generate matrix of Reed Muller code (1,3)

R = m Cr => R = 3C1 = 3C1 + 3C0 = 4

M = 2m => M = 23 = 8

5. Generate a Reed Muller Code

In this paper Reed Muller code is generated with the help of Hadamard code. So before the generate matrix first we familiar with Hadamard code .

1. HADMARD CODE: The Hadamard Code Generator block generates a Hadamard code from a Hadamard matrix, whose rows form an orthogonal set of codes. Orthogonal codes can be used for spreading in communication systems in which the receiver is perfectly synchronized with the transmitter. In these systems, the dispreading operation is ideal, as the codes are de-correlated completely.

$$H_{22} = \begin{bmatrix} H_{11} & H_{11} \\ H_{11} & \overline{H_{11}} \end{bmatrix}$$

$$H_{11} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \qquad H_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Only if 4 divides n. Since HnHtn = n In, any two different rows of Hn must be orthogonal, and the matrix obtained from the permutation of rows or columns of Hn is also a Hadamard matrix, but the symmetry may be lost. Clearly -Hn is also a Hadamard matrix.

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These matrices were introduced by Jacques Hadamard in 1893. And yet, despite much attention of numerous mathematicians, the central question of existence has not been answered: we do not know whether or not, for every integer m, there is an orthogonal 4m by 4m matrix of plus and minus ones; this, notwithstanding that the number of such matrices seems to grow extremely rapidly with m, the combinatorial explosion coming perhaps as early as m = 7. Still less is known about the classification of Hadamard matrices for general m; but they have been enumerated for m < 7.

The Hadamard Code Generator block outputs a row of HN. The output is bipolar. You specify the length of the code, N, by the **Code length** parameter. The **Code length** must be a power of 2. You specify the index of the row of the Hadamard matrix, which is an integer in the range [0, 1, ... , N-1], by the **Code index** parameter.

Generate matrix in form of reed Muller coding: this can be.

$$H_{11} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \qquad H_{2,2} = H_{1,1} \otimes H_{1,1}$$

$$H_{22} = \begin{bmatrix} H_{11} & H_{11} \\ H_{11} & \overline{H}_0 \end{bmatrix}$$

understood with the help some example.

Example: Generate reed Muller code of (1, 3) with the help Hadamard code.

We know the Hadamard code has **nxn** form. So assume this **3x3** matrix so.

Length ofthe vectorr is **2^m = 8**

$$H_{33} = \begin{bmatrix} H_{11} & H_{11} & H_{11} & H_{11} \\ H_{11} & H_{00} & H_{11} & H_{00} \\ H_{11} & H_{11} & H_{00} & H_{00} \\ H_{11} & H_{00} & H_{00} & H_{00} \end{bmatrix}$$

$$H_{33} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

OR

$$H_{33} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

But here the requirement the rows and Colum's are

R = ${}^{m}C_r$ => R = ${}^{3}C_1 = {}^{3}C_1 + {}^{3}C_0$ = 4

M = 2^3 => M = 23 = 8

From the above question the requirement of row for reed Muller code is 4 but the Hadamard code matrix has 8-rows and 8- column. So we reduce the row as per the system requirement. And the problem is how to reduce the row from Hadamard to Reed Muller. For reduction follow the following steps.

- Reduction of rows is calculated by **Re = $2^{(M - R)}$** $= 2^{(3-1)} = 4$
- Counting in the each row one by one from the Hadamard code matrix. If the total number one is greater the Re the leave that row. If the number of one is less the Re than remove than row from the Hadamard matrix.

From the above Hadamard matrix, number of rows is less the Re in row v_3, v_5, v_6 v_7. After the reduction of the Hadamard matrix the final Generate matrix of Reed Muller cod is.

$$RM_{(2,3)} \begin{bmatrix} u_0 \\ u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$RM_{(2,3)} \begin{bmatrix} u_0 \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Similarly we can generate the RM $_{(2, 4)}$, RM $_{(2, 4)}$

$$RM_{(2,3)}\begin{bmatrix} u_0 \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \\ u_9 \\ u_{10} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & & \end{bmatrix}$$

6. Conclusion

This paper examines the Reed-Muller codes is one bit error correction or detection method. The code is developed by hardened code and examine how they are used in practice in Matlab. Design some representation of generator matrices for these codes with the help of Hadamard code.

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8. Biblography

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